Engineering Notes

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Frequency-Dependent Wave Damping in the Time Domain

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ONE of the more troublesome problems associated with the study of the dynamics of floating bodies is accurate specification of wave damping effects. This is particularly true for arbitrary shapes, such as floating platforms or buoys, for which there are little experimental data or theoretical results available. Strip theory has been used to theoretically predict wave damping for simple shapes, such as cylinders and some hull shapes. Experimental measurements of wave damping have also been made for several hull shapes.[1] For arbitrary shapes, however, there is little theory available and in the absence of experimental data, only an estimation can usually be made.

The most common estimation of the wave-damping coefficients in heave $d_{\dot{z}}$ and pitch $d_{\dot{\theta}}$ for floating, three-dimensional bodies are obtained by Haskind's equations[1,2]

$$d_{\dot{z}} = \left(\frac{1}{2g}\right)\rho\omega^3\alpha^2L^2B^2\chi^2K_z \tag{1}$$

$$d_{\dot{\theta}} = \left(\frac{1}{4g^3}\right)\rho\omega^7J_y{}^2\chi^2K_\psi \tag{2}$$

In these equations ρ = mass density of water, ω = wave frequency, α = waterplane coefficient, L = length, B = beam, g = gravitational constant, and J_y = pitch moment of inertia of the waterplane area. The correction coefficient χ corrects for the Smith effect and is a function of the draft to wavelength ratio. The coefficients K_z and K_ψ are length correction coefficients and are functions of the length to wavelength ratio. These approximate formulations of the damping coefficients provide an easily obtained estimation for wave-damping coefficients in heave and pitch. They are, however, wave-frequency dependent which leads to difficulties in multi-frequency or random waves when a nonlinear dynamic model for heave or pitch is used. It is the purpose of this Note to report the time domain analogs of Haskind's equations which permit analysis of heave and pitch in cases for which transform techniques cannot be used.

For the case of small platforms in which the characteristic length L is small in comparison with the wavelength λ, specifically $\pi L/\lambda \ll 1$, further approximation of Eqs. (1) and (2) is permissible. Under this condition, which is frequently the case, the length correction coefficients are essentially unity. In addition, a good fit of the Smith effect is given by the experimental function

$$\chi = e^{-C_vH\omega^2/g} \tag{3}$$

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where H = draft and C_v = vertical coefficient. Using the definition of the waterplane coefficient, $\alpha = A_w/LB$, and incorporating the above conditions, Eqs. (1) and (2) become

$$d_{\dot{z}} = (1/2g)\rho\omega^3A_w{}^2e^{-2C_vH\omega^2/g} \tag{4}$$

$$d_{\dot{\theta}} = (1/4g^3)\rho\omega^7J_y{}^2e^{-2C_vH\omega^2/g} \tag{5}$$

where A_w = waterplane area.

Consideration of the pure heave or pure pitch of a floating platform with the inclusion of frequency-dependent wave damping leads to an ordinary differential equation of the form

$$m\ddot{x} + \int_0^t D_{\dot{x}}(t-\tau)\dot{x}(\tau)d\tau + Q_{\dot{x}}(\dot{x},t) + K(x) = F(t) \tag{6}$$

In this equation m = actual plus constant hydrodynamic added mass, x = measure of motion under consideration (z, heave; θ, pitch; $x(t) = 0$; $t < 0$), $Q_{\dot{x}}(\dot{x},t)$ = nonlinear damping, $K(x)$ = buoyancy restoring force, and $F(t)$ = wave force excitation. The nonlinear damping term $Q_{\dot{x}}$ arises from profile drag or skin friction drag, both of which are quadratic functions of the relative velocity between the platform and the wave. The buoyance restoring force, although frequently taken to be linear, is commonly a nonlinear function of the displacement x for many platforms or buoys.

The convolution integral in Eq. (6) arises because of the input (wave) frequency dependence of the coefficient $d_{\dot{x}} = d_{\dot{z}}$ or $d_{\dot{\theta}}$.[3,4] The time function $D_{\dot{x}}$ is the function whose Fourier cosine transform yields $d_{\dot{z}}$ in the case of heave or $d_{\dot{\theta}}$ in the case of pitch. In the time domain, the frequency-dependent damping appears as "damping with memory" and takes the form of convolution. It is assumed that the system is causal or realizable, which leads to the integral limits 0 and t.

For the case of a linearized model of the dynamics, $Q_{\dot{x}} = 0$ and $K(x) = kx$, where k is a constant. Under these conditions use of the Fourier transform readily produces the output spectrum. The convolution integral presents no problem since its Fourier transform is $j\omega d_x(\omega)$, where $d_{\dot{x}}$ is given by Eq. (4) or (5) and the time function $D_{\dot{x}}(t)$ is not required to be known. In addition, because the system is linear, superposition can be used for the response with multi-frequency or random wave inputs.

When the nonlinear model is considered the situation is markedly different. The Fourier transform technique is, of course, no longer useful. If standard approximations, such as equivalent linearization or perturbation techniques, are not acceptable or tractable then numerical solution of Eq. (6) is required. Numerical solution demands that the convolution integral be treated and, consequently, that the time function $D_{\dot{x}}(t)$ be known.

Consider the second-order differential equation

$$\beta^2\ddot{y} + t\dot{y} + (n+1)y = 0 \tag{7}$$

where β^2 is a constant, n is an integer, $y(t) = 0$ for $t < 0$, and the initial conditions are chosen to be $y(0) = y_0, \dot{y}(0) = 0$. Taking the Fourier transform yields the following first-order differential equation

$$-\omega(dY/d\omega) + [n - \beta^2\omega^2]Y = j\beta^2\omega y_0 \tag{8}$$

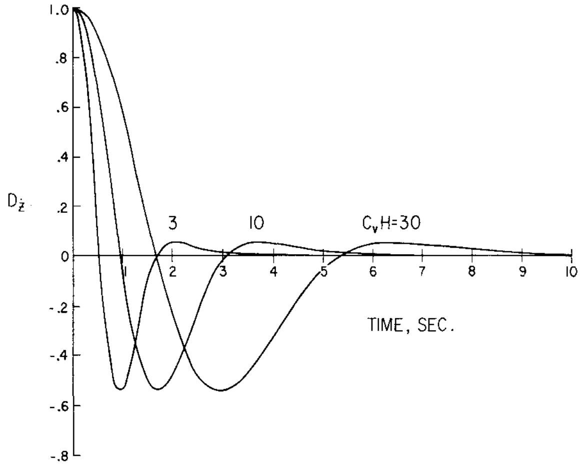

Fig. 1 Wave damping function for heave.

The solution for the real part of $Y(j\omega)$ is

$$Y_R = C|\omega|^n e^{-\beta^2\omega^2/2} \tag{9}$$

where the constant C is given by

$$C = \frac{\pi[\beta/(2)^{1/2}]^{n+1}}{[(n-1)/2]!} y_0 \tag{10}$$

If the constant β^2 is taken to be $4C_vH/g$, then the exponential in Eq. (9) is in agreement with the exponential in Haskind's equations. For wave damping in heave, $n = 3$ and $y_0 = 2\rho A_w^2/(\pi g\beta^4)$ makes $Y_R(\omega)$ identically equal to $d_{\dot z}$ given by Eq. (4). Similarly, $n = 7$ and $y_0 = 24\rho J_y^2/(\pi g^3\beta^8)$ makes $Y_R(\omega)$ identical to the pitch damping coefficient $d_{\dot\theta}$ given by Eq. (5).

Thus for the case of nonlinear heave or nonlinear pitch models, the time function $D_{\dot x}(t)$ required in the convolution integral in Eq. (6) can be provided by solution of a differential equation of the type given by Eq. (7). The even part of the transform of this solution gives the desired wave-damping, frequency-dependent coefficient. For the case of heave, the differential equation required is

$$(4C_vH/g)\ddot y + t\dot y + 4y = 0 \tag{11}$$

For the case of pitch,

$$(4C_vH/g)\ddot y + t\dot y + 8y = 0 \tag{12}$$

In these equations the dependent variable y is associated with and scaled to be the required function $D_{\dot x}$ appearing in Eq. (6).

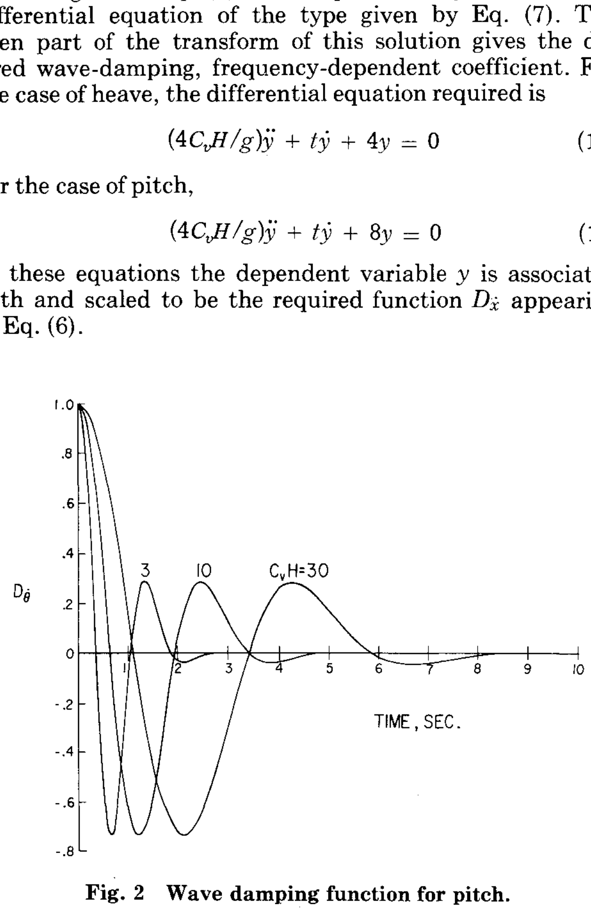

Fig. 2 Wave damping function for pitch.

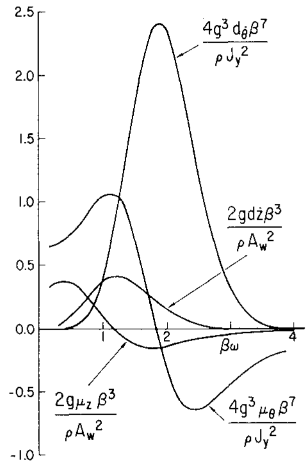

Fig. 3 Frequency-dependent damping and added mass.

Solutions of Eqs. (11) and (12) are given in terms of confluent hypergeometric functions[5] and as such are quite inconvenient. For numerical studies however, Eqs. (11) and (12) may be numerically integrated a priori and their solutions stored in the computer for use in the simulation of Eq. (6). Alternately, either equation can simply be adjoined to Eq. (6) and simultaneous numerical solution of both may proceed.

The solutions of Eqs. (11) and (12) for values of C_vH of 3, 10, and 30, and for the initial conditions $y(0) = y_0 = 1$, $\dot y(0) = 0$ are shown in Figs. 1 and 2. The wave-damping function in heave makes only one negative loop before exponentially decaying to zero. The pitch function makes two negative loops. The smaller the parameter C_vH, (i.e., the smaller the draft or the larger the waterplane area) the shorter the time required for the function to decay. This is equivalent to a larger bandwidth in the frequency domain.

The imaginary parts of the Fourier transforms of the solutions to Eqs. (11) and (12) are related to the frequency-dependent parts of the added mass coefficients in heave and pitch. Specifically, the frequency-dependent added mass coefficient is given by

$$\mu_x(\omega) = -1/\omega \int_0^\infty D_{\dot x}(t)\sin\omega t\,dt \tag{13}$$

These functions, for both heave and pitch, are shown in Fig. 3 along with the damping coefficients. Although the added mass functions appear to be reasonable, Ogilvie[4] has pointed out that such functions derived from damping approximations may be unreliable particularly at extreme values of frequency.

In summary, studies of heave and pitch dynamics utilizing nonlinear models and frequency-dependent wave damping can be accomplished by consideration of an additional linear, time-varying differential equation. The solution of this equation provides the time function required for the damping convolution.

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